PROBLEM 15.

Optical Tunnelling

PROBLEM

Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected.

EXPERIMENT

- Two measurement ranges:
  - Centimeter waves – accurate measuring
  - Visile light – obtaining the effect
- Parameters:
  - Waveelngth of the light used
  - Refraction index of prisms and medium in the gap
  - Polarization
  - Distance between prisms

1. CENTIMETER WAVES

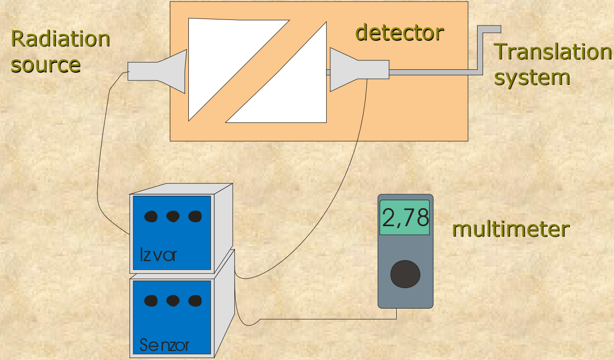
- Wavelength: 3 cm
- Polarization: linear, electrical field perpendicular to plane of propagation
- Prism refraction index: 1.5 (paraffin)
- Measurements:
    - Intensity of tunneled waves
    - Intenzity of reflected waves

in dependence on prism distance

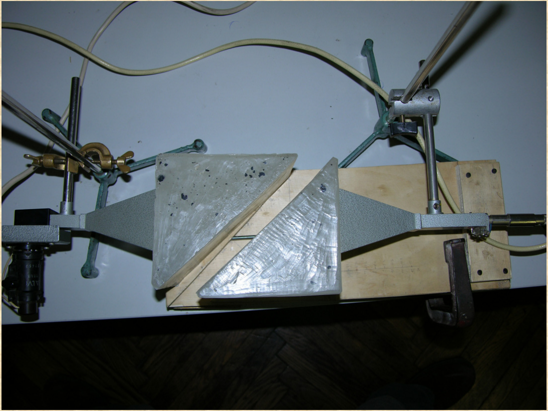
- Measured – voltage in the detector
- Voltage – proportional to field!

CENTIMETER WAVES cont.

- Apparatus schematic:

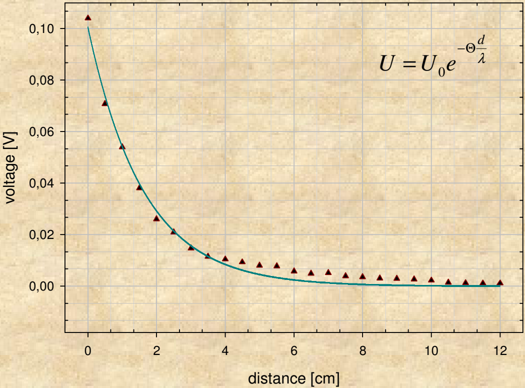


CENTIMETER WAVES cont.



CENTIMETER WAVES cont.

Tunelled field:



2. VISIBLE LIGHT

- Wavelenght: 780 nm
- Polarization: linear
    - electric field perpendicular to plane of propagation
- Prism index of refraction: 1.48 (measured)
- Measurements:
    - Intensity of tunneled waves
    - Intensity of reflected waves
- Intensity measurement: photodiode
- Voltage – proportional to square of field!

2. VISIBLE LIGHT

- Measurement in time
  - A slow motor (0.5 r/min) moves the translator
  - Voltage sampling at the diode every 1/50 of a second
- The signal grows in time
- Change of prism distance:

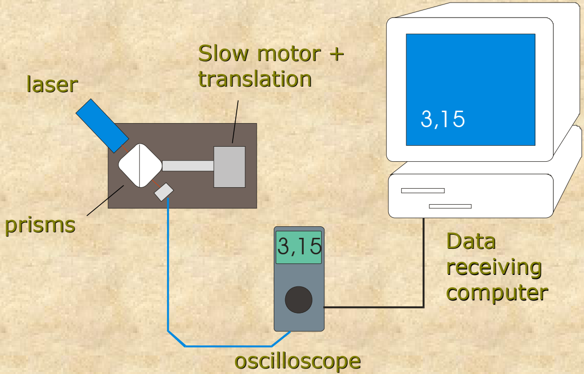
$$\Delta d = -vt$$

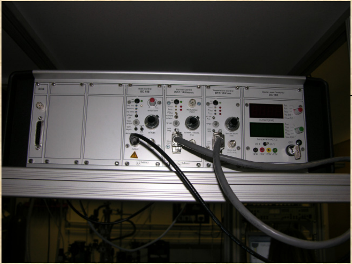
v – translator speed

t – elapsed time

VISIBLE LIGHT cont.

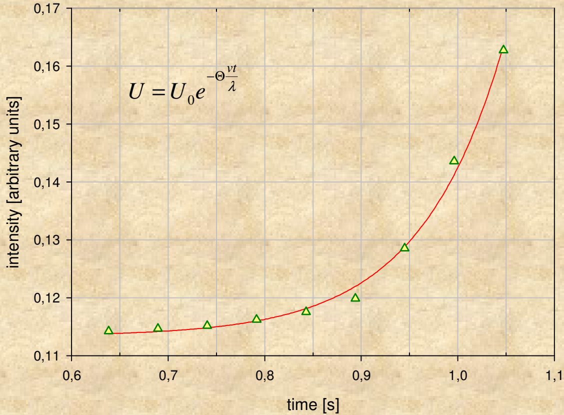
- Apparatus schematic:




DCB
Beam Control
SC 100
amplitude
offset
Current Control
DCC 100
modulation input
Temperature Control
DTC 100
Diode Laser Controller
DC 100
current (mA)
temperature (°C)
OFF


DANGER
DL 100 L
TuiOptics
Ser.Nr.:
Art.Nr.:
CE

VISIBLE LIGHT cont.



EXPLANATION

- Huygens principle:

Every atom ˝through˝ which light passes is a source of light identical to the incident light

⇒ Electromagnetic waves in dielectrics – the resultant of interference of the initial wave and all scattered waves

EXPLANATION cont.

- At total reflection – the reflected ray is the only interference maximum
- Behind the reflection plane – destructive interference, but only *far away* from the plane
- Close to the plane (distances of the order of the wavelength) the waves haven’t interfered completely and a decaying field exists

EXPLANATION cont.

- That field decays fast due to interference
- If a prism is put into the field – a new interference maximum can be formed in the prism
- A new, tunnelled wave is formed in the prism
- The energy of the reflected wave becomes smaller

MAXWELL EQUATIONS

$$\nabla \mathbf{E} = -\frac{1}{\varepsilon_0}\nabla \mathbf{P}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \mathbf{B} = 0$$

$$c^2 \nabla \times \mathbf{B} = \frac{1}{\varepsilon_0}\frac{\partial \mathbf{P}}{\partial t} + \frac{\partial \mathbf{E}}{\partial t}$$

E – electric field

B – magnetic field induction

P **–** polarization

c – speed of light in a vacuum

ε_0 – vacuum permittivity

PLANE WAVE SOLUTIONS

Electrical field:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\omega t - \mathbf{kr})}$$

Magnetic field:

$$\mathbf{B} = \frac{1}{\omega}\mathbf{k} \times \mathbf{E}$$

$\mathbf{E}_0$ – amplitude

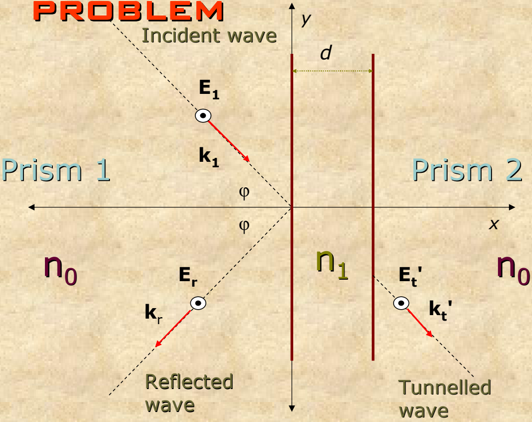
ω – frequency

t – time

k – wave vector

r - radiusvector

GEOMETRY OF THE PROBLEM



BOUNDARY CONDITIONS

- If the electric field is perpendicular to the wave vector plane:

$$E_{10} + E_{r0} = E_{t0}$$

E_{10}, E_{r0}, E_{t0}– amplitudes of the incident, reflected and transmitted waves

k_{1x}, k_{rx}, k_{tx}– x components of the wave vectors of the incident, reflected and transmitted waves

$$k_{1x}E_{10} + k_{rx}E_{r0} = k_{tx}E_{t0}$$

BOUNDARY CONDITIONS cont.

- For the wave vectors:

$$k_{ty} = k_{1y}$$

$$k_{tx}^2 = \left(\frac{n_1}{n_0}\right)^2 k_1^2 - k_{1y}^2$$

k_{1y}, k_{ty}– y components of the incident and transmitted wave vectors

n_0 – prism index of refraction

n_1 – medium between prisms index of refraction

SOLUTION

- If the incident angle is greater than the reflection angle, Snell's law gives

$$k_{tx} = ik_1 \frac{n_1}{n_0}\sqrt{\left(\frac{n_0}{n_1}\right)^2 \sin^2\varphi - 1}$$

φ – incident angle

- *x* – component of the wave vector is a pure imaginary => the wave propagates along the plane

=> the amplitude decays exponentially

SOLUTION cont.

- The field in the second prism:

$$E_t' = E_{10} e^{-2\pi n_1 \frac{d}{\lambda}\sqrt{\left(\frac{n_0}{n_1}\right)^2 \sin^2\varphi - 1}} = E_{10} e^{-\Theta\frac{d}{\lambda}}$$

d – prism distance

λ – vacuum wavelegth of incident light

Θ – decay coefficient

COMPARATION – decay coefficient

- Centimeter waves:

Experimental	2.5±0.1
Theoretical	2.22

- Optical range:

Experimental	1.1±0.1
Theoretical	1.94

COMPARATION cont.

- Agreement is relatively good
- Error causes:
  - Imprecise prism refraction index values
  - In optical range:
    - Prism surface defects and dust
    - Motor precision ...

CONCLUSION

- We have obtained, measured and modelled optical tunnelling
- It may be said:

The only condition for light incident on a prism plane with an angle greater than the critical angle not reflecting completely is to put another prism plane next to the original plane to a distance of the order of the wavelength used