Problem 14.

Einstein – De Haas Experiment

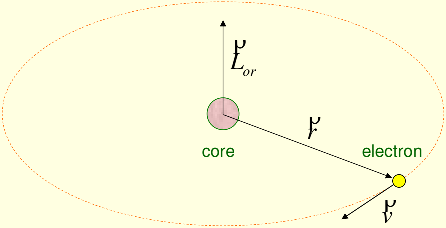
Explanation - introduction

- Explanation outline:
    - Spin and orbital moments of electrons
    - Relation to magnetic moment
    - External field influence
    - Transfer to lattice
    - Angular momentum conservation

Electron mechanical moments

Two moments:

1. Orbital => ″revolution″ around core

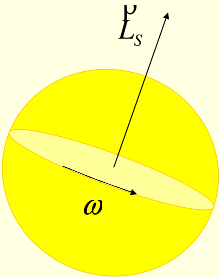


$$L_{or} = mvr$$

m – electron mass

Electron mechanical moments cont.

2. Spin => quantum property - ˝rotating electron˝ (semiclassical picture)



$$L_S = J\omega$$

J – moment of inertia of a spherical electron:

$$J = \frac{2}{5} m R^2$$

m – electron mass

R – electron radius

Electron magnetic moments

- The electron is charged – the magnetic moments are linked to magnetic moments

=> A magnetic field can change the mechanic moment of an electron system!

Orbital moment:

$$\mu_{or} = \frac{1}{2} evr$$

v – electron velocity

r – trajectory radius

Spin moment (classically):

$$\mu_s = -\frac{2}{5} eR^2 \omega$$

ω – rotation angular velocity

R– electron radius

Electron magnetic moments cont.

⇒ Important (and measurable) qunatities: gyromagnetic ratios

$$\boxed{\Gamma_{or} = \frac{e}{2m}}$$

$$\boxed{\Gamma_s = -\frac{e}{m}}$$

Γ_{or} – orbital moment ratio

Γ_s – spin moment ratio

m – electron mass

e – electron charge

- In a ferromagnetic – only spin moments!

External field influence

- Torque on electron in external magnetic field:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

μ – electron magnetic moment

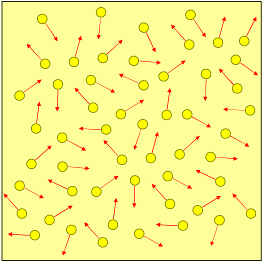
B - field

⇒ Dipoles turn into field directon - MAGNETIZATION

- Thermal motion opposes the field influence
- Magnetization – equilibrium between magnetic and thermal motion

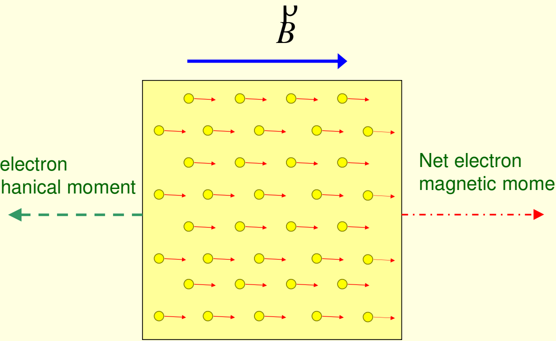
Field influence cont.

- Dipole ordering causes a change in the net mechanical moment of the eelctrons:



No field

Field influence cont.



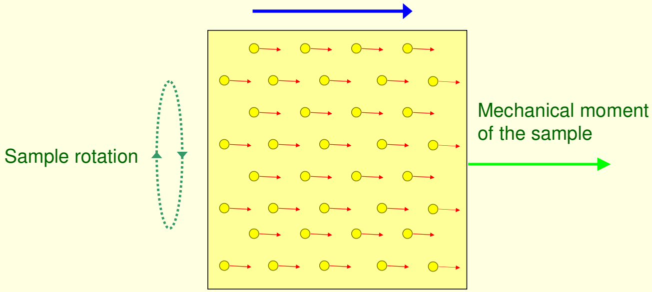
Net electron
mechanical moment

Net electron
magnetic moment

Field on

Field influence cont.

- Due to angular momentum conservation:



Field on

Experiment - introduction

- Measurements:
  - Sample properties
  - Sample magnetization
  - Resonant frequency of the torsional oscillations
  - Damping coefficient of the thread
  - Resonant amplitude

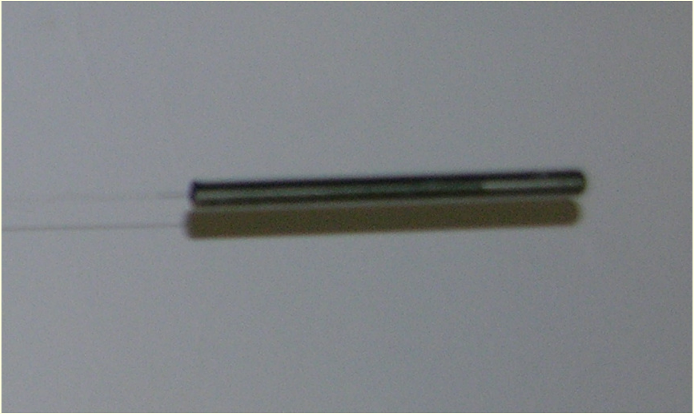
The sample

- The sample was made of soft iron because of narrow hysteresis
- Properties:

radius	0,4 cm
length	11,4 cm
density	7800 kg/m^3

- The sample was suspended on a thread of torsion $6.92 \cdot 10^{-6}$ kgm^2/s

The sample cont.



Sample magnetization

- Inductive measurement
- Around the sample – a small solenoid
- Magnetization is not linear in the magnetic field and can be written as a Fourier series:

$$M = \sum_{i=1}^{\infty} M_i \cos \omega_i t$$

M_i – amplituda *i* – tog harmonika

ω_i –frekvencija *i* – tog harmonika

- Close to resonance – harmonics neglected! (exp. verified)
- Separate harmonics measurement – Lock - in

Sample magnetization cont.

- Voltage of the *i* – th harmonic – linear to the magnetization time change:

$$U_{i_{\max}} = N r^2 \pi \mu_0 M_i \omega_i$$

N – number of detector windings

r – sample radius

μ_0 – vacuum permeability

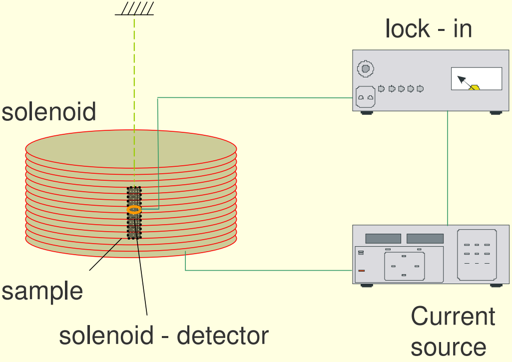
M_i – amplitude of the *i* – th harmonic

ω_i –frequency of the *i* – th harmonic

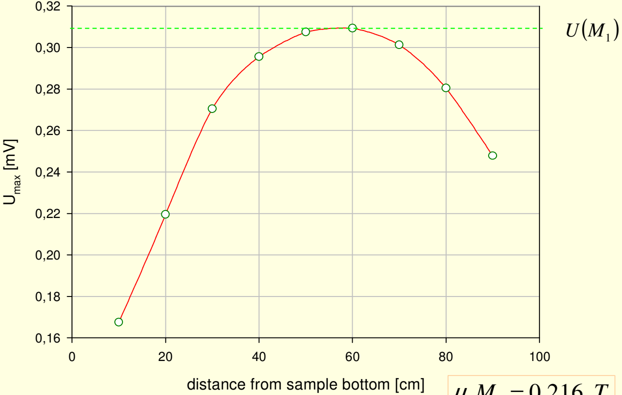
- Magnetization is approximately homogeneous (from position dependence)

Sample magnetization cont.

- Apparatus:



Sample magnetization cont.



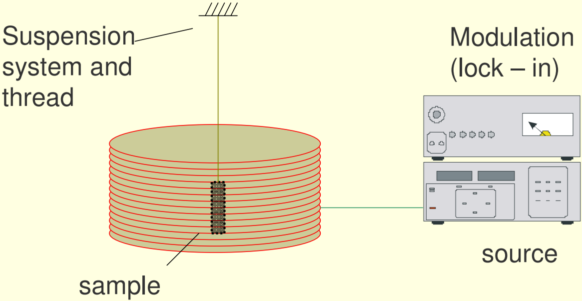
$$\mu_0 M_1 = 0.216\ T$$

Resonant frequency

- The sample was suspended on a string of small damping
- Resonance was determined by watching the oscillation amplitudes
- The amplitude is greatest in resonance
- Driving current – modulated DC source
- Resonant frequency error 10^{-2} Hz
- Value for thread A: 1.54 Hz

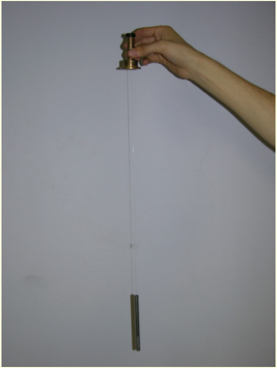
Resonant frequency cont.

- Apparatus:



Resonant frequency cont.

Suspension system



sample + thread

Damping

- Damping determined by amplitude time dependence
- Amplitude drops exponentially:

$$\varphi = \varphi_0 e^{-\frac{\alpha}{2}t}$$

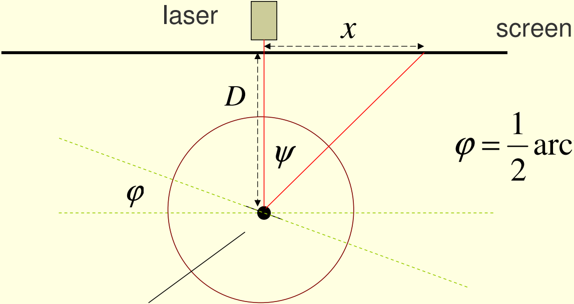
φ – angular amplitude

α – damping coefficient

t - time

- The amplitude was measured with a laser – mirror system:

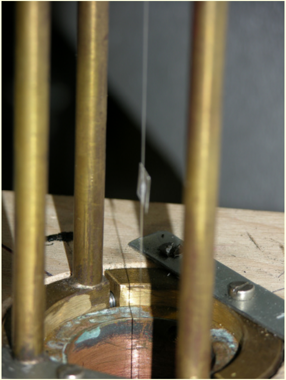
Damping cont.



Sample and mirror

$$\varphi = \frac{1}{2}\arctan\frac{x}{D}$$

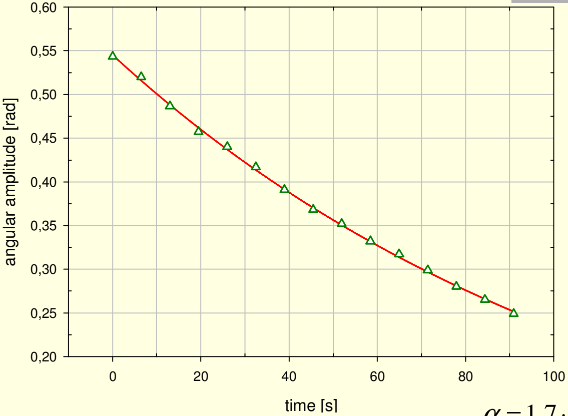
Damping cont.



Mirror on the thread

The screen

Damping cont.



$$\alpha = 1.7 \cdot 10^{-2}\ s^{-1}$$

Resonant amplitude

- Torsional oscillation equation with magnetization driving torque:

$$\ddot{\varphi} + \alpha\dot{\varphi} + \omega_0^2\varphi = -\frac{V}{\Gamma I} M_1 \omega \sin \omega t$$

α – damping coefficient

ω_0 – resonant frequency

V – sample volume

Γ – gyromagnetic ratio

I – sample moment of inertia

M_1 – magnetization amplitude

ω – driving frequency

t - time

Resonant amplitude cont.

- Resonant amplitude:

$$\boxed{\varphi_0 = -\frac{VM_1}{\Gamma \alpha J}}$$

V – volumen uzorka

M_1 – amplituda magnetizacije

Γ – giromagnetski omjer

α – koeficijent gušenja

J – moment inercije uzorka

- The gyromagnetic moment can be obtained from the measured amplitude
- Amplitude measurement – laser - mirror system

Resonant amplitude cont.

Resonant amplitude cont.

- Measured amplitude: 0,418 rad

$$\boxed{\Rightarrow \Gamma = 5.09 \cdot 10^{8}\ C/kg}$$

- The calculated value of e/m is $1.76 \cdot 10^{11}$ C/kg!
- Possible reasons of disagreement:
    - Wrong damping and amplitude measuring
    - Nonlinearity of the thread
    - Mode locking...

Spin magnetic moment

- Spin moment – classical rotating electron model:

$$d\mu = \frac{\omega}{2} r^4 \rho \pi dz \quad \Rightarrow \quad \mu = -\frac{2}{5} e R^2 \omega$$

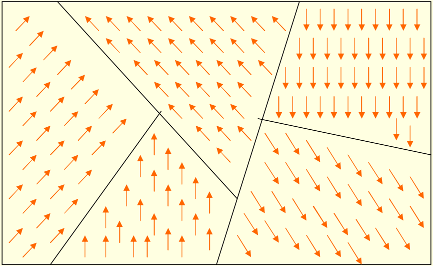
ω – angular velocity

r – electron layer radius

ρ – charge density

Ferromagnetic structure

- Domain structure (the domains contain millions of dipoles):



Ferromagnetic structure cont.

- Cause:
    - Quantum spin interactions (all in the same direction!)
    - Minimalization of energy (separationg into domains)
- The orbital moments cancel out
- Net magnetic moment of a sample – only spin moments!